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All but one free ride when wealth effects are small

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Abstract Quasilinear preferences on a public good and a numeraire good are limits of preferences where both goods are normal. The set of equilibria of the voluntary contribution (or private provision) game is easily characterized under quasilinearity by: top valuators aggregately contribute their common stand-alone contribution, whereas non-top valuators contribute nothing. Because, as long as preferences are randomly selected, there will typically be a single top valuator, it follows that, typically, the equilibrium is unique, with all players but one contributing nothing, hence "free riding" in the sense of the ordinary English usage of the expression. The upper-hemicontinuity of the Nash equilibrium correspondence implies that this is also the case when both goods are strictly normal, but the wealth effects on the public good are small.

Keywords Free riding · Public goods · Voluntary contributions · Private provision · Normal goods · Quasilinear preferences · Wealth effects

JEL Classification H41 · C72 · D70

1 Free riding

The terms *free rider* and *free riding* are frequently used both in economics and in ordinary English speech. "Free rider" has been present in economics at least since Buchanan (1964) with the following meaning: the *free-rider problem* is defined as the inefficiency of the outcome of a scheme to supply a public good by voluntary

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contribution. Hence, the free rider problem is particular case of the inefficiency of *laissez-faire* in the presence of externalities. A “free rider” in this context is somebody who does not take account of the effects of her actions on other people. Note that every participant in the usual game of voluntary contributions to public goods is a free rider in this sense, because even if a person does contribute a positive amount, her contribution would be larger if she took into account the positive externality that she generates. Because inefficiency is bad, “free riding” has a clear negative connotation in the economics discourse.

The meaning is somewhat different in the common English usage of the expression: a “free rider” is somebody who benefits from something without paying. The Oxford English Dictionary attests the term “ride” from 1886 the sense of a “rider” as “A passenger, esp. one using public transport.” “Free rider” appears in the legal literature at the turn of the 19th century in reference to not-paying passengers in trains, both with and without the authorization of the railroad, and in the 1930’s for hitchhikers. The term did not necessarily have a negative connotation in its earlier usage: it could perfectly apply to a VIP who was a guest of the railroad company.[1]

Here I use “free rider” in the sense of the ordinary English usage, i.e., as a person who contributes nothing. Granted, the discussion is framed by the voluntary contribution game where the equilibrium is indeed inefficient. But my focus is the characterization of who pays and who does not, rather than efficiency.

2 The model

Let there be a finite number of players, numbered $i = 1, \dots, I$, and let I also denote the set of players. For $i = 1, \dots, I$, Player i’s utility function is denoted $u_i : \Re_+ \times M_i \to \Re : (y, m_i), \mapsto u_i(y, m_i)$, where $M_i \subset \Re$, y is the amount of the public good, and $m_i \in M_i$ is the amount of a private, numeraire good. Player i is endowed with ω_i units of numeraire, and decides on her *contribution* t_i towards the supply of the public good. We require t_i to be nonnegative. Given a tuple $(t_1^*, \dots, t_I^*)$, we define the supply of the public good as $t^* \equiv \sum_{h \in I} t_h^*$ and, for $i \in I$, we write $t_{-i}^* \equiv \sum_{\substack{h \in I \\ h \neq i}} t_h^*$.

A *Nash equilibrium of the voluntary contribution* (*or private provision*) *game* is a tuple $(t_1^*, \dots, t_I^*) \in \Re_+^I$ of contributions such that, for each $i \in I$, t_i^* maximizes $u_i(t_{-i}^* + t_i, \omega_i - t_i)$ subject to $t_i \geq 0$.

Let $(t_1^*, \dots, t_I^*)$ be an equilibrium. We say that i is a *contributor* at $(t_1^*, \dots, t_I^*)$ if $t_i^* > 0$. If, on the contrary, $t_i^* = 0$, then we say that, at $(t_1^*, \dots, t_I^*)$, i is a *non contributor* or that i *free rides*.

3 Normality

It is well known that when both the public good and the private good are normal (positive wealth effects) the following properties hold (Warr 1983; Bergstrom et al. 1986, 1992; Andreoni 1988; Fraser 1992; Buchholz et al. 2006a,b).

[1] See Silvestre (2008), where the difficulties in translating the “free rider” into other languages are discussed.

(1) *Uniqueness* (Bergstrom et al. 1986, Theorems 2–3): There is a unique Nash equilibrium, which implies a unique tuple of consumption vectors.
(2) *Warr neutrality* (Warr 1983; Bergstrom et al. 1986): A redistribution of wealth among contributors does not change the equilibrium consumption vectors.

4 Quasilinearity

4.1 Assumptions

Quasilinearity assumption. $M_i = \Re$ and $u_i(y, m_i) = v_i(y) + m_i$, for some function $v_i : \Re_+ \to \Re$.

Remark 1 Postulating $M_i = \Re$ guarantees the global absence of wealth effects (see, e.g., Mas-Colell et al. 1995). Alternatively, as it is frequently done, one can assume that $M_i = \Re_+$ and that ω_i is "high enough," which here requires ω_i to exceed i's "stand-alone contribution," defined below.

We may variously refer to $v_i(y)$ as i's *valuation of*, *benefit from*, or *willingness to pay for* y units of the public good. Accordingly, when $v_i(y)$ is differentiable, $v_i{\prime}(y)$ is the marginal valuation of (or benefit from, or willingness to pay for) the public good at y. Note that if the free disposal of the public good is postulated, then v_i cannot be decreasing anywhere.

Single peakedness assumption. The function $b_i : \Re_+ \to \Re : b_i(y) = v_i(y) - y$ is single peaked, i.e., $\exists \hat{y}_i \geq 0$ such that

$$\begin{array}{ll} b_i(\hat{y}_i) > b_i(y) & \text{for all } y \geq 0, \\ \text{if } y^0 < y^1 < \hat{y}_i, & \text{then } b_i(y^1) > b_i(y^0), \\ \text{if } \hat{y}_i < y^0 < y^1, & \text{then } b_i(y^0) > b_i(y^1). \end{array}$$

Under the single-peakedness assumption we call $\hat{y}_i$ i's *stand-alone contribution*.

Note that the single peakedness assumption holds if and only if b_i is strictly quasi-concave and has a maximizer.

The single-peakedness assumption is satisfied by most, if not all, valuation functions used in the modeling of the private provision game. It is in particular satisfied in the popular case where v_i is strictly concave, continuous at $y = 0$ and bounded. It is also satisfied when v_i is continuous, increasing and strictly concave on an interval $[0, \bar{y}]$ and constant on $(\bar{y}, \infty)$: the amount $\bar{y}$ is then a satiation level of the public good, and a free disposal postulate makes the valuation function constant above $\bar{y}$.

Given an profile of I quasilinear preferences, define the set of *top valuators* as $T \equiv \{i \in I : \hat{y}_i \geq \hat{y}_h, \forall h \in I\}$, and the *top valuation* as $\hat{y} \equiv \hat{y}_i$, for any $i \in T$. Quasilinearity implies that whether i is a top valuator or not depends only on the profile of preferences, and not on the vector of individual endowments of the numeraire good.

4.2 Characterization of the Nash equilibrium

Proposition *Under quasilinearity and single peakedness* $(t_1^*, \ldots, t_1^*)$ *is a Nash equilibrium if and only if it is nonnegative and satisfies*

(a) $t^* = \hat{y}$;
(b) $i \notin T \Rightarrow t_i^* = 0$.

Proof (Straightforward; offered for self-containment.) We first show that (a) and (b) imply that $(t_1^*, \ldots, t_1^*)$ is a Nash equilibrium. Suppose not, i.e., there is a player i and a $t_i \geq 0$ such that $v_i(t_{-i}^* + t_i) + \omega_i - t_i > v_i(t^*) + \omega_i - t_i^*$, i.e.,

$$v_i(t_{-i}^* + t_i) + \omega_i - t_{-i}^* - t_i > v_i(t^*) + \omega_i - t_{-i}^* - t_i^*. \tag{1}$$

Because, by (a), t^* is the stand-alone contribution of a top valuator, (1) cannot hold for $i \in T$.

So let $i \notin T$, which implies that i′s stand-alone's contribution is strictly less than the stand-alone contribution of a top valuator, i.e.,$\hat{y}_i < \hat{y} = t^* = t_{-i}^*$ (by (a) and (b)). Therefore $\hat{y}_i < t_{-i}^* \leq t_{-i}^* + t'$, yet $b_i(t_{-i}^* + t_i) > b_i(t_{-i}^*)$ (by (1)), contradicting the single peakedness of b_i.

Conversely, let $(t_1^*, \ldots, t_1^*)$ be a Nash equilibrium. To prove (a), assume, by contradiction, that $t^* \neq \hat{y}$. If $t^* < \hat{y}$, then choose a top valuator i, in which case $v_i(\hat{y}) - \hat{y} > v_i(t^*) - t^*$, i.e., $v_i(t_{-i}^* + \hat{y} - t_{-i}^*) + t_{-i}^* - \hat{y} > v_i(t_{-i}^* + t_i^*) + t_{-i}^* - t_i^*$, or writing $t_i \equiv \hat{y} - t_{-i}^*$, we have that $t_i > 0$ (because $\hat{y} > t^* \geq t_{-i}^*$) and that $v_i(t_{-i}^* + t_i) - t_i > v_i(t_{-i}^* + t_i^*) - t_i^*$, contradicting the assumption that $(t_1^*, \ldots, t_1^*)$ is a Nash equilibrium. If $t^* > \hat{y}$, then let i be any player for whom $t_i^* > 0$. Define $\varepsilon \equiv \min\{t_i^*, t^* - \hat{y}_i\}$, positive because $\hat{y}_i \leq \hat{y} < t^*$, and consider $t_i \equiv t_i^* - \varepsilon \in [0, t_i^*)$, i.e., $\hat{y}_i \leq t_{-i}^* + t_i$ and $t_{-i}^* + t_i < t_{-i}^* + t_i^*$. By single peakedness, $v_i(t_{-i}^* + t_i) - (t_{-i}^* + t_i) > v_i(t_{-i}^* + t_i^*) - (t_{-i}^* + t_i^*)$, i.e., $v_i(t_{-i}^* + t_i) + \omega_i - t_i > v_i(t_{-i}^* + t_i^*) + \omega_i - t_i^*$, contradicting the assumption that $(t_1^*, \ldots, t_1^*)$ is a Nash equilibrium. This proves (a).

To prove (b), let $i \notin T$ and $t_i^* > 0$. Again, define $\varepsilon \equiv \min\{t_i^*, t^* - \hat{y}_i\}$, positive because $\hat{y}_i < \hat{y} = t^*$(by (a)), and proceed as in the previous paragraph. □

Remark 2 The result is intuitive. Let $v_i(y)$ be differentiable with $v_i''(y) < 0$ (or differentiable on $[0, \bar{y}]$ with $v_i''(y) < 0$ on $[0, \bar{y})$ and $v_i'(y) = 0$ on $(\bar{y}, \infty)$. Then $b_i'(y) \equiv v_i'(y) - 1$ is i's *marginal net valuation of her contribution* and $\hat{y}_i$ is defined by $b_i'(\hat{y}_i) \leq 0$ with $b_i'(\hat{y}_i)\hat{y}_i = 0$. Let $\hat{y} > 0$. If i is a top contributor, then $b_i'(\hat{y}) = 0$, whereas if not, then $b_i'(\hat{y}) < 0$. But these are essentially the Kuhn–Tucker conditions of the Nash equilibrium.

4.3 Single and multiple top valuators

It follows from the Proposition that, in the *single-top-valuator case* (the set T is a singleton), there is a unique Nash equilibrium. Provided that preferences are randomly selected among a rich enough set of quasilinear preferences, this is the typical situation. There is then a single contributor, and everybody else free rides.

If, on the contrary, and more exceptionally, we are in the *multiple-top-valuator* case, then the equilibrium contributions of top valuators are undetermined as long as they add up to $\hat{y}$.

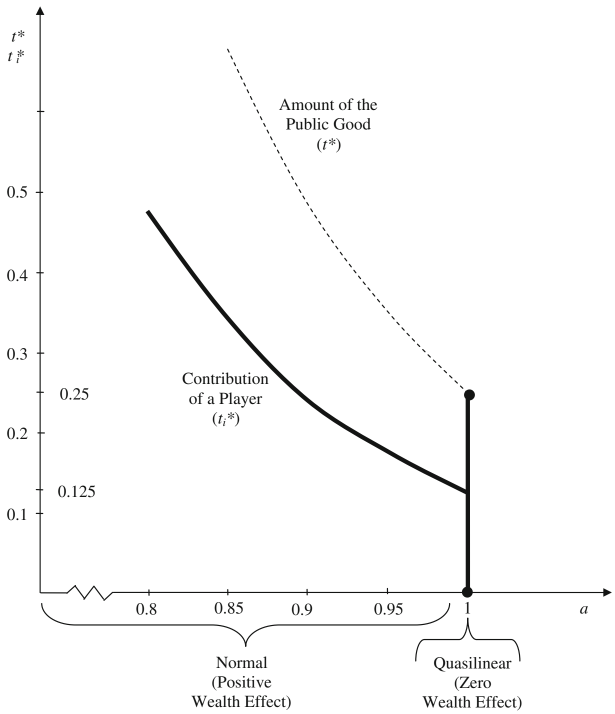

Fig. 1 The equilibrium correspondence as the wealth effect tends to zero. Example 1: Two players with identical preferences

5 Comparison between normality and quasilinearity

Quasilinear preferences are limits of preferences where both goods are normal as the wealth effects on the public good tend to zero. It is therefore instructive to compare the properties of the model with positive wealth effects, listed in Sect.3 above, with the corresponding ones in the limiting, quasilinear case.

(1) *Uniqueness.* The amount of the public good is uniquely determined both under normality and quasilinearity. But the equilibrium allocation, unique under normality, displays a degree of indeterminacy in the quasilinear, multiple-top-valuator case. This implies that, if we take a sequence of profiles of normal preferences that converge to a quasilinear, multiple-top-valuator profile of preferences, the equilibrium correspondence is not lower hemicontinuous. Example 1 illustrates: two identical individuals with $\omega_i = 10$, and a sequence of utility functions of the form $\sqrt{y} + m_i^a$, $i = 1, 2$, where the sequence of $a\prime$s is increasing and tends to one. Figure 1 graphs the equilibrium correspondence: the thick line indicates the contribution of one of the players, whereas the dotted line indicates the amount of the public good.

(2) *Warr neutrality.* In the single-top-valuator case, Warr neutrality trivially holds. In the multiple-top-valuator case, the set of equilibrium private-good consumptions is no longer invariant with respect to wealth redistributions among top valuators. But a version of Warr neutrality holds, namely that of Theorem 1 in

Bergstrom et al. (1986), stating that after the redistribution, there is a new Nash equilibrium in which every player changes the amount of her contribution by precisely the change in her wealth.

6 Free riding at positive wealth effects

The form of free riding present in the quasilinear, single-top-valuator case, where all but one player contribute nothing, does not necessarily occur with normal preferences, possibly excepting large numbers of players or widely different preferences (see Andreoni 1988; Buchholz et al. 2006a). But it does typically occur when the wealth effects on the public good are positive but small.

Indeed, consider a sequence of differentiable, strictly quasiconcave normal economies converging to a quasilinear economy with a single top valuator. If i is a non-top-valuator at the limit, then $b_i'(y) < 0$ there (Remark 2), and, because of the upper-hemicontinuity of the equilibrium correspondence (Fudenberg and Tirole 1991; see also Kohlberg and Mertens 1986), it must be the case that $\frac{\partial u_i/\partial y}{\partial u_i/\partial m_i} - 1 < 0$ as the

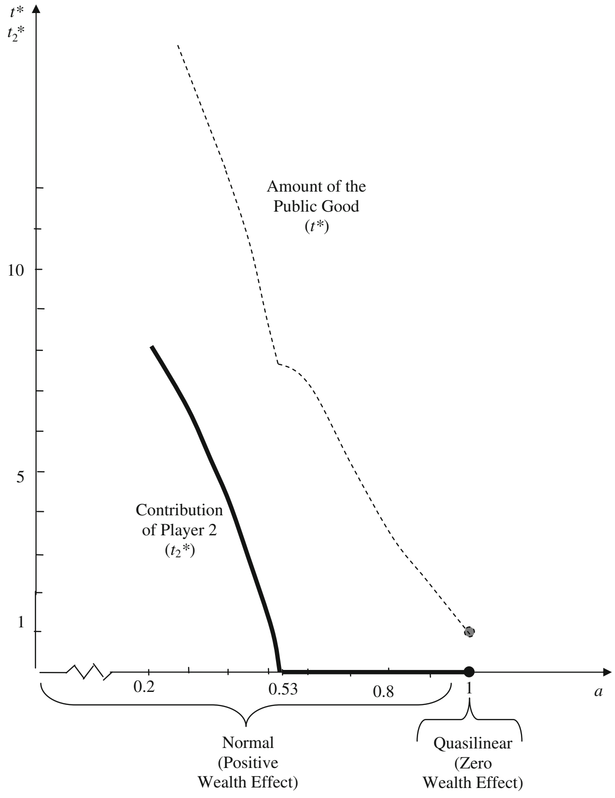

Fig. 2 The equilibrium correspondence as the wealth effect tends to zero: Example 2: Two players with different preferences

sequence approaches its limit and wealth effects on the public good are positive but small. Hence, there will be a single contributor for small enough wealth effects.

This is a general observation, which applies to any number of players, and any nonzero degree of preference differentiation. Example 2 illustrates: two players, each endowed with 10 units of numeraire. Player 1's utility function is $2\sqrt{y} + m_1^a$, and Player 2's is $\sqrt{y} + m_2^a$. Figure 2 depicts the equilibrium correspondence as a tends to one: For high enough a (in Example 2, for $a > 0.53$), Player 1 is the only contributor, and Player 2 free rides.

7 Conclusion

When the wealth effects on the public good are small, as long as preferences are randomly selected, the presumption is that only one player contributes, whereas all the other players contribute nothing to the provision of the public good. The behavior of the non contributing players constitute "free riding" in the strong sense that the expression has in ordinary English usage. This result is independent from the number of players: it applies to both small and large numbers.

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